Two-Way ANOVA

Two-way ANOVA

- So far, our ANOVA problems had only one dependent variable and one independent variable (factor). (e.g. compare gas mileage across different brands)
- What if want to use ***two or more independent variables?*** (e.g. compare gas mileage across different brands of cars and in different states)
- We will only look at the case of two independent variables, but the process is the same for larger number of independent variables.
- When we are examining the effect of two independent variables, this is called a **Two-Way ANOVA.**

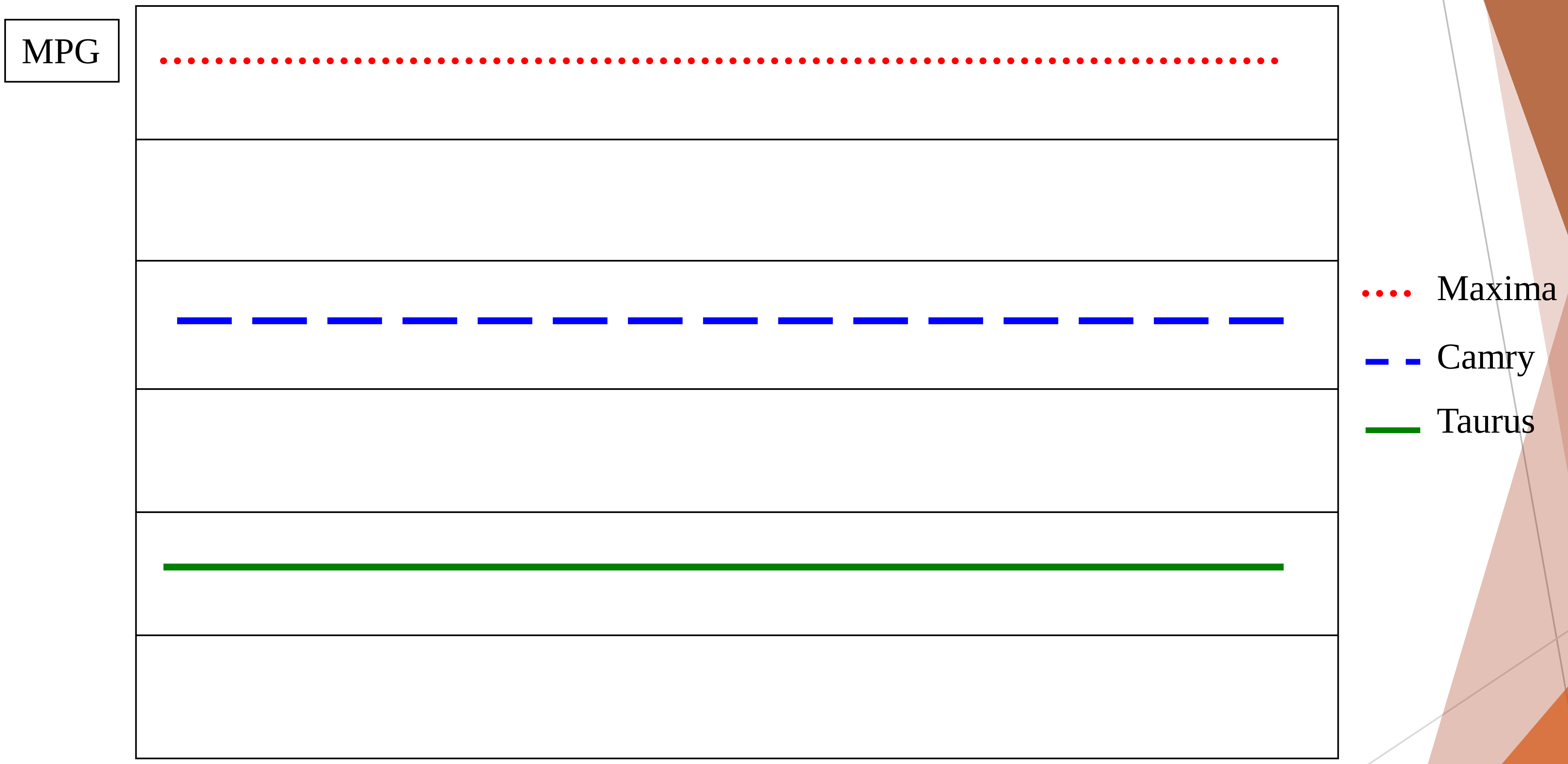
Two-way ANOVA

- In a **Two-way ANOVA**, the effects of two factors can be investigated *simultaneously.*
- Two-way ANOVA permits the investigation of the effects of either factor alone (e.g. the effect of brand of car on the gas mileage, and the effect of the state on the gas mileage) and also the *two factors together* (e.g.the combined effect of the model of the car and the effect of state on gas mileage).
- This ability to look at both factors together is the advantage of a Two-Way ANOVA compared to two One-Way ANOVA's (one for each factor)

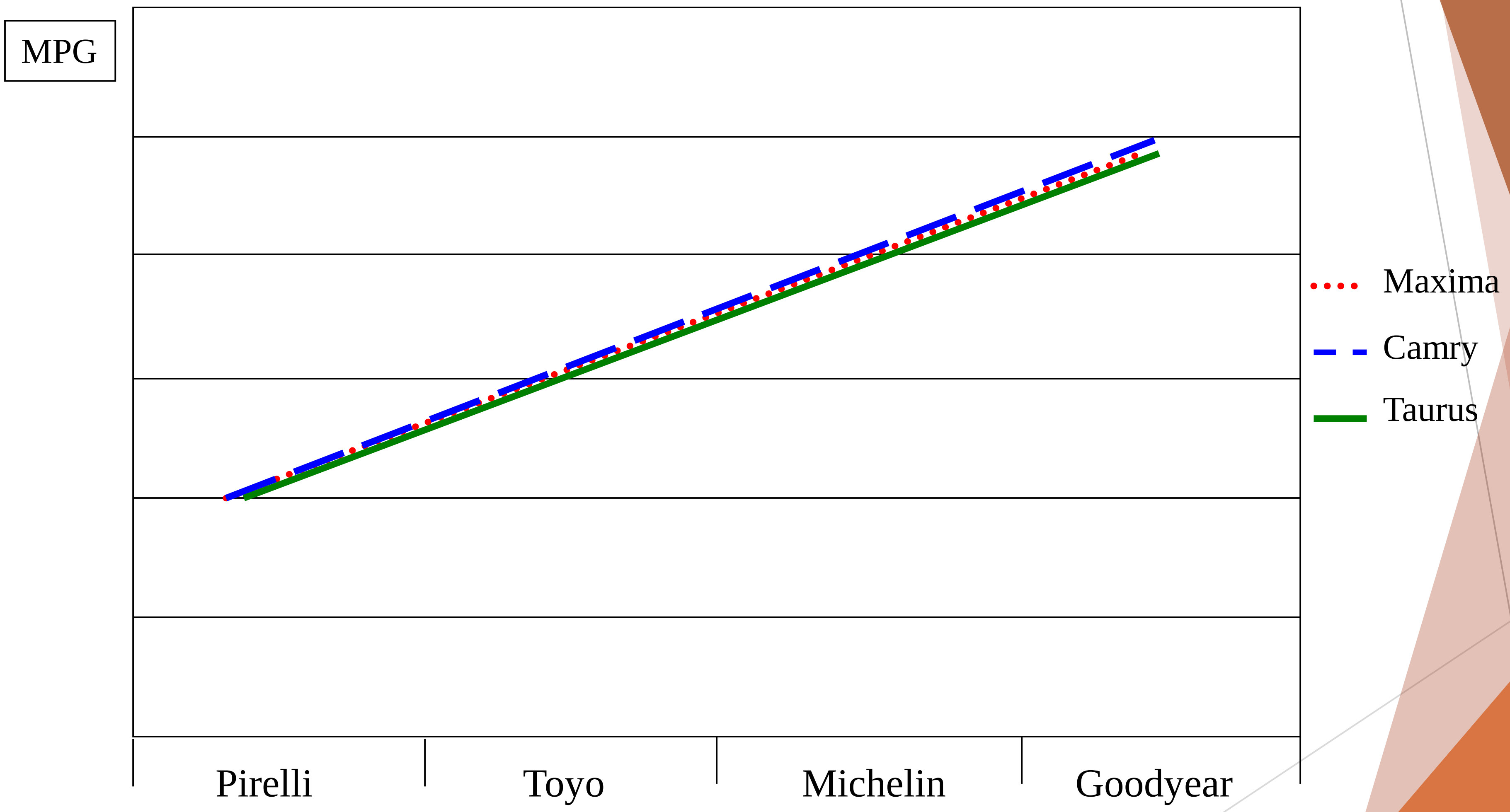
Two-way ANOVA

- The effect on the population mean (of the dependent variable) that can be attributed to the levels of either factor alone is called a **main effect.** This is what you would detect using two separate one-way ANOVA's.

Main Effect of Car Brand



Main Effect of Tire Brand



Hypotheses

- Two questions are answered by a Two-way ANOVA
  - Is there any effect of Factor A on the outcome? ***(Main Effect of A)***.
  - Is there any effect of Factor B on the outcome? ***(Main Effect of B)***.

- This means that we will have two sets of hypotheses, one set for each question.

Hypotheses

1) Main effect of Factor A:

H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \ldots \alpha_a = 0$ or, $\alpha_i = 0$ for all i = 1 to a
(a = # of levels of A)

H_1: not all α_i are 0 or, at least one $\alpha_i \neq 0$

2) Main effect of Factor B:

H_0: $ß_1 = ß_2 = ß_3 = \ldots ß_b = 0$ or, $ß_j = 0$ for all j = 1 to b
(b = # of levels of B)

H_1: not all $ß_j$ are 0 or, at least one $ß_j \neq 0$

Hypotheses

Effect of brand of car (Factor A) and tire (Factor B) on gas mileage (dependent variable).

1) Main effect of Car Brand:

H_0: There is no difference in average gas mileage across different brands of cars.

H_1: There are differences in average gas mileage across different brands of cars.

2) Main effect of Tire Brand:

H_0: There is no difference in average gas mileage across different brands of tires.

H_1: There are differences in average gas mileage across different brands of tires.

Sum Squares

- In One-way ANOVA, the relationship between the sums of squares was:

$$SST = SSTR + SSE$$

- In Two-way ANOVA, we have two factors, which means we have separate treatment levels for those two factors. Thus the relationship becomes:

$$SST = SSA + SSB + SSE$$

Where:

SSA: Variance between different levels of factor A

SSB: Variance between different levels of factor B

Mean Squares and F value

Mean Squares:

$$MSA = \frac{SSA}{(a - 1)}$$

$$MSB = \frac{SSB}{(b - 1)}$$

$$MSE = SSE/(a-1)(b-1)$$

F-calculated:

$$F_A = \frac{MSA}{MSE}$$

$$F_B = \frac{MSB}{MSE}$$

Two Way ANOVA Table

Source of Variation	Sum of Squares	df	Mean Square	F-ratio	F-critical
Factor A	SSA	a-1	MSA= SSA /(a-1)	F= MSA/MSE	F [(a-1),(a-1)(b-1)]
Factor B	SSB	b-1	MSB= SSB /(b-1)	F= MSB/MSE	F [(b-1),(a-1)(b-1]
Error	SSE	(a-1)(b-1)	MSE= SSE /(a-1)(b-1)		
TOTAL	SST	ab-1			

a- Number of treatment levels (categories) for Factor A.

b- Number of treatment levels (categories) for Factor B.

TWO WAY ANOVA EXAMPLE

- A group of students are interested in testing how popular it is to watch Olympic events at Vancouver. They wonder if there is an effect of five different events or the day of week that the events are scheduled for (Friday, Saturday, or Sunday). The results are analyzed with a two-way ANOVA Table shown below:

Source of Variation	Sum of Squares	df	Mean Square	F-ratio	F-critical
Event	568				
Day	63				
Error	170				
TOTAL	801				

Two Way ANOVA with Interactions Hypotheses

- Three questions are answered by a Two-way ANOVA with Interactions

  - Is there any effect of Factor A on the outcome?

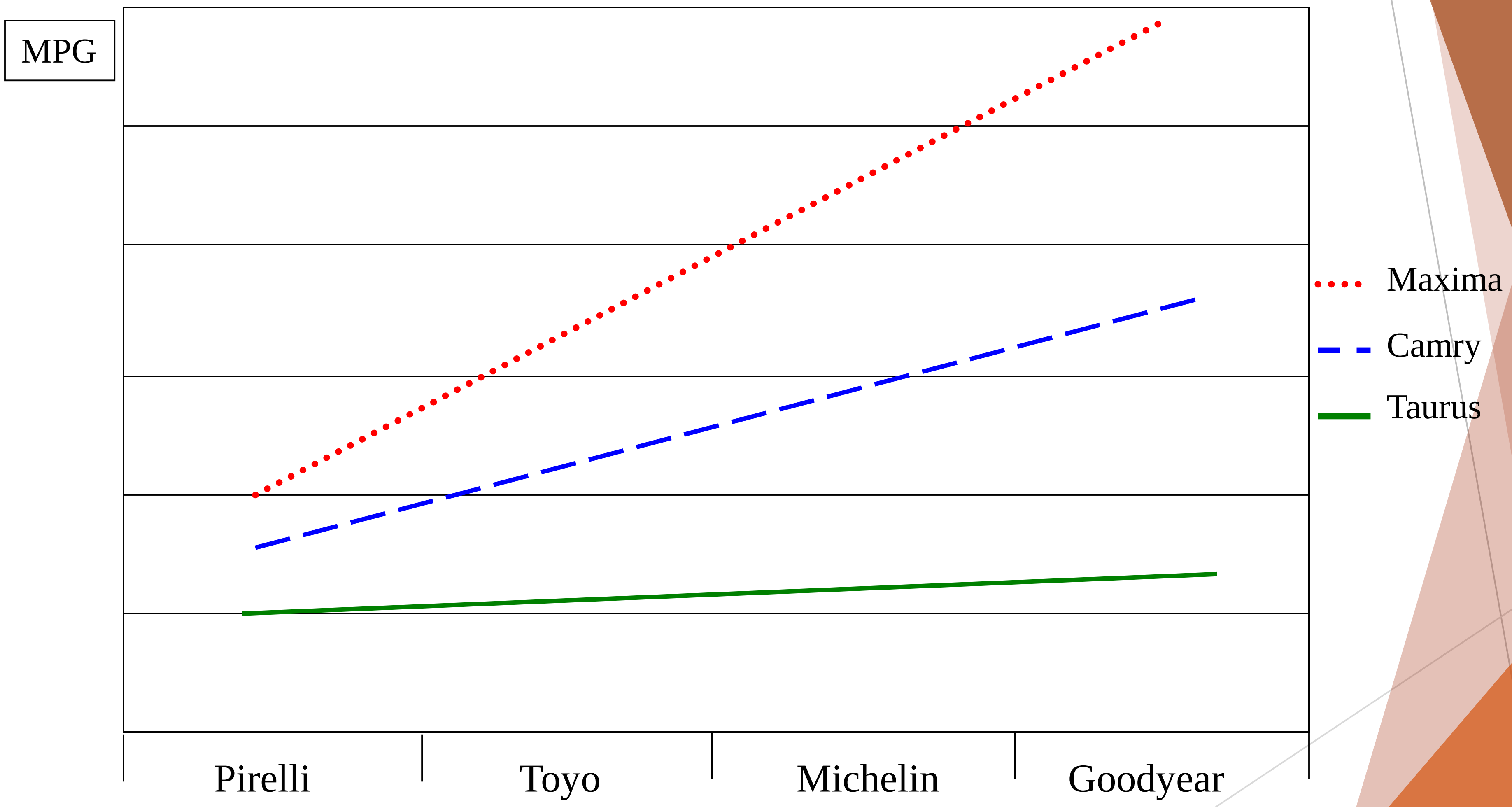
    (Main Effect of A).
  - Is there any effect of Factor B on the outcome?

    (Main Effect of B).
  - Is there any effect of the interaction of Factor A and Factor B on the outcome?

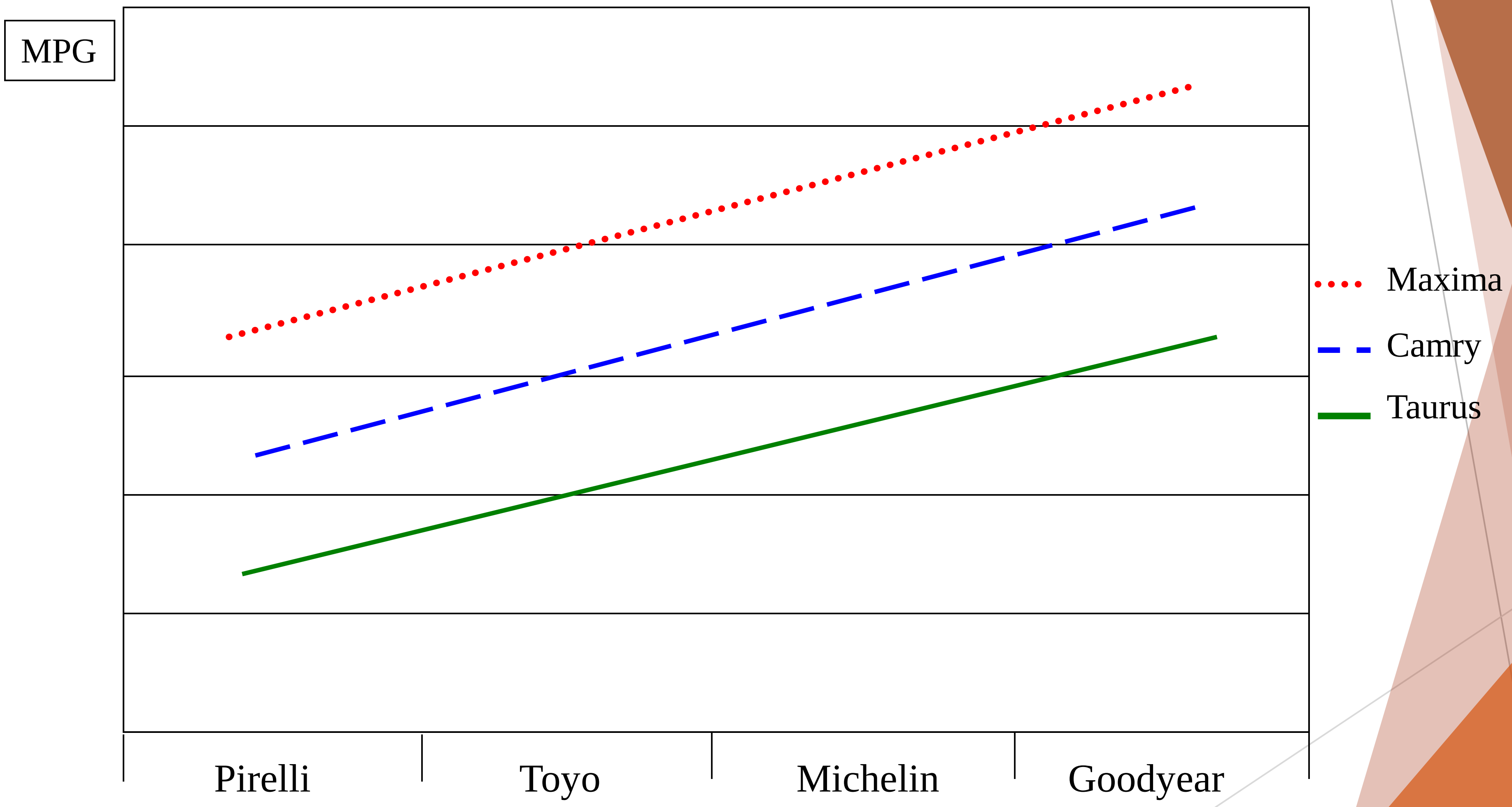
    (Interactive Effect of AB)

- This means that we will have three sets of hypotheses, one set for each question.

Interaction Effect of Car and Tire



NO Interaction Effect of Car and Tire



Hypotheses

1) Main effect of Factor A:

H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \ldots \alpha_a = 0$ or, $\alpha_i = 0$ for all i = 1 to a
(a = # of levels of A)
H_1: not all α_i are 0 or at least one $\alpha_i \neq 0$

2) Main effect of Factor B:

H_0: $ß_1 = ß_2 = ß_3 = \ldots ß_b = 0$ or, $ß_j = 0$ for all j = 1 to b
(b = # of levels of B)
H_1: not all $ß_j$ are 0 or at least one $ß_j \neq 0$

3) Interactive Effect of AB:

H_0: $\alpha ß_{ij} = 0$ for all i and j
H_1: the $\alpha ß_{ij}$ are not all 0

Two Way ANOVA with Interactions Table

Source of Variation	Sum of Squares	df	Mean Square	F-ratio	F-critical
Factor A	SSA	a-1	MSA= SSA / (a-1)	F= MSA/MSE	F [(a-1),ab(n-1)]
Factor B	SSB	b-1	MSB= SSB / (b-1)	F= MSB/MSE	F [(b-1),ab(n-1)]
Interaction	SSAB	(a-1)(b-1)	MSI = SSAB / (a-1)(b-1)	F= MSAB/MSE	F [(a-1)(b-1),ab(n-1)]
Error	SSE	ab(n-1)	MSE= SSE / ab(n-1)		
TOTAL	SST	abn-1			

a: Number of treatment levels (categories) for Factor A.

b: Number of treatment levels (categories) for Factor B.

n: number of observations per cell

Mean Squares and F value

Mean Squares:

$$MSA = \frac{SSA}{(a-1)}$$

$$MSB = \frac{SSB}{(b-1)}$$

$$MSAB = \frac{SSAB}{(a-1)(b-1)}$$

$$MSE = \frac{SSE}{ab(n-1)}$$

F-calculated:

$$F_A = \frac{MSA}{MSE}$$

$$F_B = \frac{MSB}{MSE}$$

$$F_{AB} = \frac{MSAB}{MSE}$$

Two-Way ANOVA with Interaction Example

A group of students are interested in testing how popular it is to watch Olympic events at Vancouver. They wonder if there is an effect of five different events or the day of week that the events are scheduled for (Friday, Saturday, or Sunday). A total of 75 students are surveyed so that each combination cell of event and day has an equal number of students. The results are analyzed with a two-way ANOVA with interactions table shown below:

Source of Variation	Sum of Squares	df	Mean Square	F-ratio	F-critical
Event	133				
Day	63				
Interaction					
Error	486				
TOTAL	801				

Two-Way ANOVA with Interaction Example 2

The Neilson Company is interested in testing for differences in average viewer satisfaction with morning news, evening news, and late news. The company is also interested in determining whether differences exist in average viewer satisfaction with the three main networks: CBS, ABC, NBC. Nine groups of 50 viewers are assigned to each combination. Complete the following ANOVA table and interpret the results.

Source of Variation	Sum of Squares	df	Mean Square	F-ratio	F-critical
Network	145				
Time	160				
Interaction	240				
Error	6,200				
TOTAL					