Training Workshop on Data Management

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Department of Medical Research (Pyin Oo Lwin branch)

Comparing means of two groups
(*t*-test)

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Objectives of this lecture

We aim participants to

1. decide when to use *t*-test
2. understand the basic concepts of *t*-distribution and types of *t*-test
3. interpret the *t*-test result

Outline

1. When to use *t*-test
2. Basic concepts of *t*-distribution and types of *t*-test
3. Interpretation and presentation of *t*-test results
4. Group exercise

Some considerations….

- How important are *t*-test?
  - Used in <u>one in three</u> papers, they are an important aspect of medical statistics.

- How easy are they to understand?
  - The details of the tests themselves are difficult to understand.

- Descriptive analysis
  - Frequency, percent, central tendency, Standard scored
- Correlational analysis
  - Correlation
  - Regression
- Analyzing differences between groups
  - ***t*-tests**
  - One-way ANOVA
  - Factorial ANOVA
  - MANOVA

…common statistical analysis (Dr Thida)

When are *t*-test used?

- Very often we are interested in comparing <u>two populations</u> in studies.
- To compare samples of “normally distributed” data.
- If the data do not follow a normal distribution, these tests should not be used.****

“TWO” ?????????

Examples:

Research Questions

1. Is the average midterm grade in “Class A” higher than the average midterm grade in Class B?
2. Is the average grade in “Mathematics” higher than “Chemistry” in this Class A?

1. Is the average midterm grade in “Class A” higher than the average midterm grade in Class B?

A

B

average midterm grade

2. Is the average grade in “Mathematics” higher than “Chemistry” in this Class A?

average grade in Mathematics

A

average grade in Chemistry

- Each sample is an example of testing a claim between two populations.
- However, there is a fundamental difference between 1) and 2).
- In # 2) the samples are not independent where as in # 1), they are.

<u>Why</u>?

- 1. Different people in each class.
- 2. Same people writing different test.

Independent Samples

- Two samples are independent if the sample selected from one population is not related to the sample selected from the other population.

Dependent Samples

- If one sample is related to the other, the samples are dependent.
- With dependent samples we get two values for each person, sometimes called paired-samples.

Example: Un-Paired Samples

samples from two independent (unrelated) groups

Participants	Male salary	Female salary
1	57000	27000
2	40200	18750
3	21450	12000
4	21900	13200
5	45000	21000
…..		

Example: Paired Samples
samples from two dependent (related) groups

Participants	Current Salary	Salary at entry
1	57000	27000
2	40200	18750
3	21450	12000
4	21900	13200
5	45000	21000
…..		

Think of your research questions! Answer…

- Do your research questions involve two samples? If yes,….

- Are they independent samples or dependent samples?

Answers..

- Group 1
  - xxx
  - xxx
- Group 2
  - xxx
  - xxx
- Group 3
  - xxx
  - xxx
- Group 4
  - xxx
  - xxx

Which Variables???

Table 5.1. Statistics for assessing an association between two variables, unpaired data

Risk factor (independent variable, exposure, group assignment)	Outcome (dependent variable)					
	Dichotomous	Nominal	Interval, normal distribution	Interval non-normal	Ordinal	Time to event, censored data
Dichotomous	Chi-squared, Fisher's exact test, risk ratio, odds ratio	Chi-squared	*t*-test	Mann-Whitney test	Chi-squared for trend, Mann-Whitney test	Log-rank, Wilcoxon, rate ratio
Nominal	Chi-squared, exact test	Chi-squared	ANOVA	Kruskal–Wallis test	Kruskal–Wallis test	Log-rank, Wilcoxon
Interval, normal distribution	*t*-test	ANOVA	Linear regression, Pearson's correlation coefficient	Spearman's rank correlation coefficient	Spearman's rank correlation coefficient	–
Interval, non-normal	Mann-Whitney test	Kruskal–Wallis test	Spearman's rank correlation coefficient	Spearman's rank correlation coefficient	Spearman's rank correlation coefficient	–
Ordinal	Chi-squared for trend, Mann-Whitney test	Kruskal–Wallis test	Spearman's rank correlation coefficient	Spearman's rank correlation coefficient	Spearman's rank correlation coefficient	–

Table 5.15. Comparison of parametric and non-parametric statistics for testing bivariate associations

	If interval variable is:	
Type of variables	Parametric	Non-parametric
Dichotomous variable and an interval variable	*t*-test	Mann-Whitney test
Nominal variable and an interval variable	ANOVA	Kruskal–Wallis test
Correction for multiple pairwise comparisons of interval variables	Bonferroni	Dunn's test
Two interval variables	Pearson's correlation coefficient, linear regression	Spearman's rank correlation coefficient

Recall memories … Variables

- Types of variables…..???????

Examples…

Example: 1

- Is the average midterm grade in "Class A" higher than the average midterm grade in Class B?
  - DV: average midterm grade
  - IV: class (A,B)

Example: 2

- Is the average grade in "Mathematics" higher than "Chemistry" in this Class A?
  - DV: average grade
  - IV: subject (M,C)

Examples…

Example: 4

- Is there any difference between the mean salary of Male and Female?
  - DV: mean salary
  - IV: gender (M,F)

Example: 5

- Is the mean of beginning salary lower than the current salary?
  - DV: mean salary
  - IV: job (B,C)

Summary!

- When *t*-test are used?
  - Continuous dependent variable
  - Categorical independent variable as two groups (dichotomous)
  - Comparing means of two samples
  - Independent samples/Dependent samples

t-distribution & t-test

- The “t” distribution (student t distribution) is a probability distribution
- similar to the standard normal (z) distribution
- used to test hypothesis involving <u>numerical data</u>
- used in estimating the mean or comparison of means of a normally distributed population <u>when sample size is small</u>

- Definition of t test
  - It’s a method of testing hypothesis about the mean of small sample drawn from a normally distributed population when the standard deviation for the sample is unknown.

- ..first published in 1908 by William Sealy Gosset,
- ..prohibited from publishing under his own name, so as pseudonym *Student*.

Recall memories …

- Normal (z) distribution?

- According to the central limit theorem, the sampling distribution of a statistic (like a sample mean) will follow a normal distribution, as long as the sample size is sufficiently large.
- Therefore, when we know the standard deviation of the population, we can compute a z-score, and use the normal distribution to evaluate probabilities with the sample mean.

- But sample sizes are sometimes small, and often we do not know the standard deviation of the population.
- When either of these problems occur, statisticians rely on the distribution of the ***t*-statistic** (also known as the ***t*-score**),

- There are actually many different t distributions.
- The particular form of the *t*-distribution is determined by its **degrees of freedom**.
- The degrees of freedom refers to the number of independent observations in a set of data.

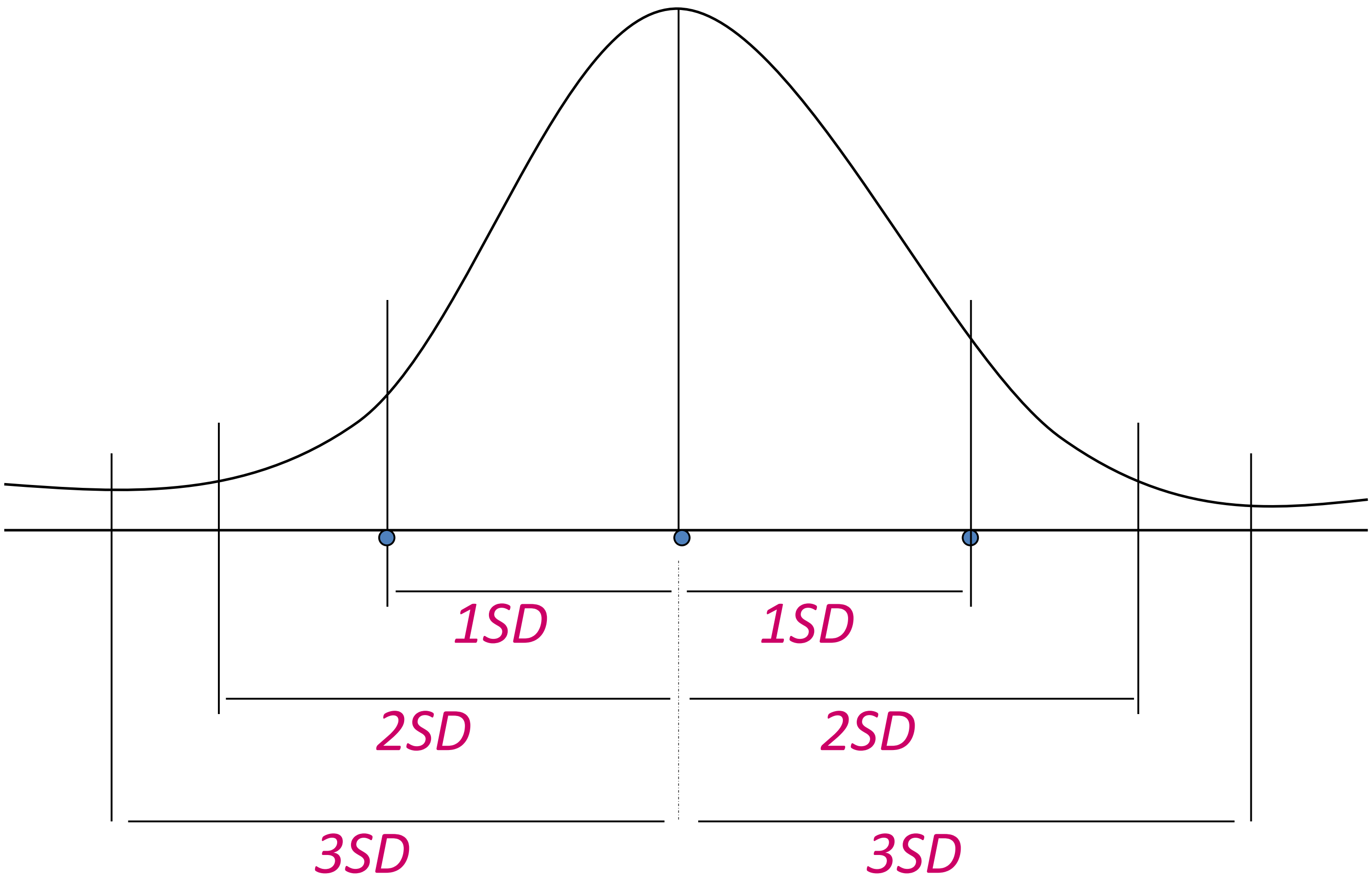
- **Properties of the t Distribution**
  - Symmetric; unimodal, bell shaped
  - $\mu = 0$
  - Looks a lot like the standard normal curve
  - Major difference: tail of t are more plump, centre is more flat
  - a family of distribution, **one for every (n)** , not actually n, but degrees of freedom

- The t distribution is flatter than the standard normal distribution and it's tails are higher and wider, indicating that it has a greater standard deviation ,especially for the small sample sizes.

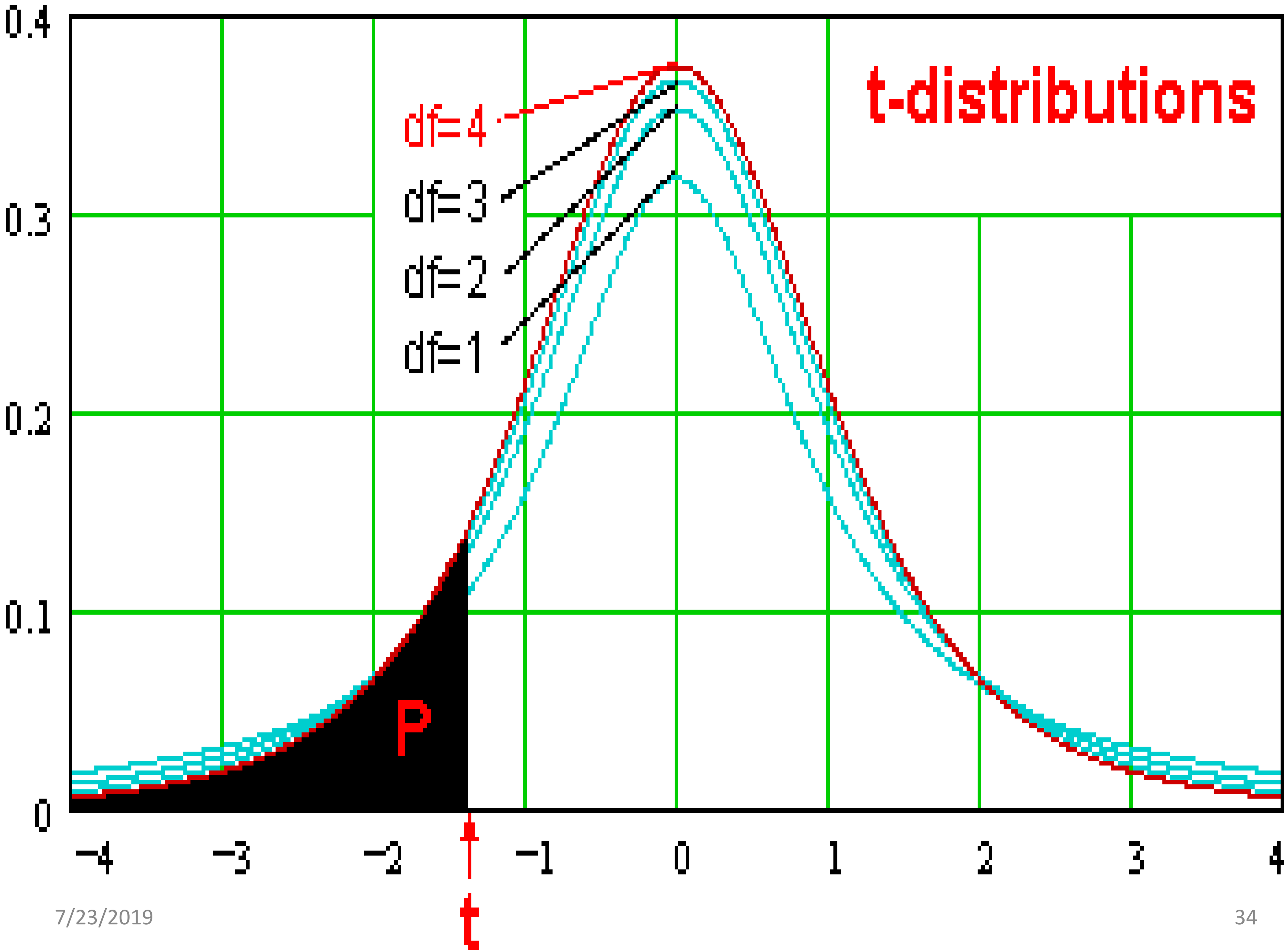
- As the sample size increases, the degree of freedom also increase; and *t*-distribution approaches the standard normal distribution.

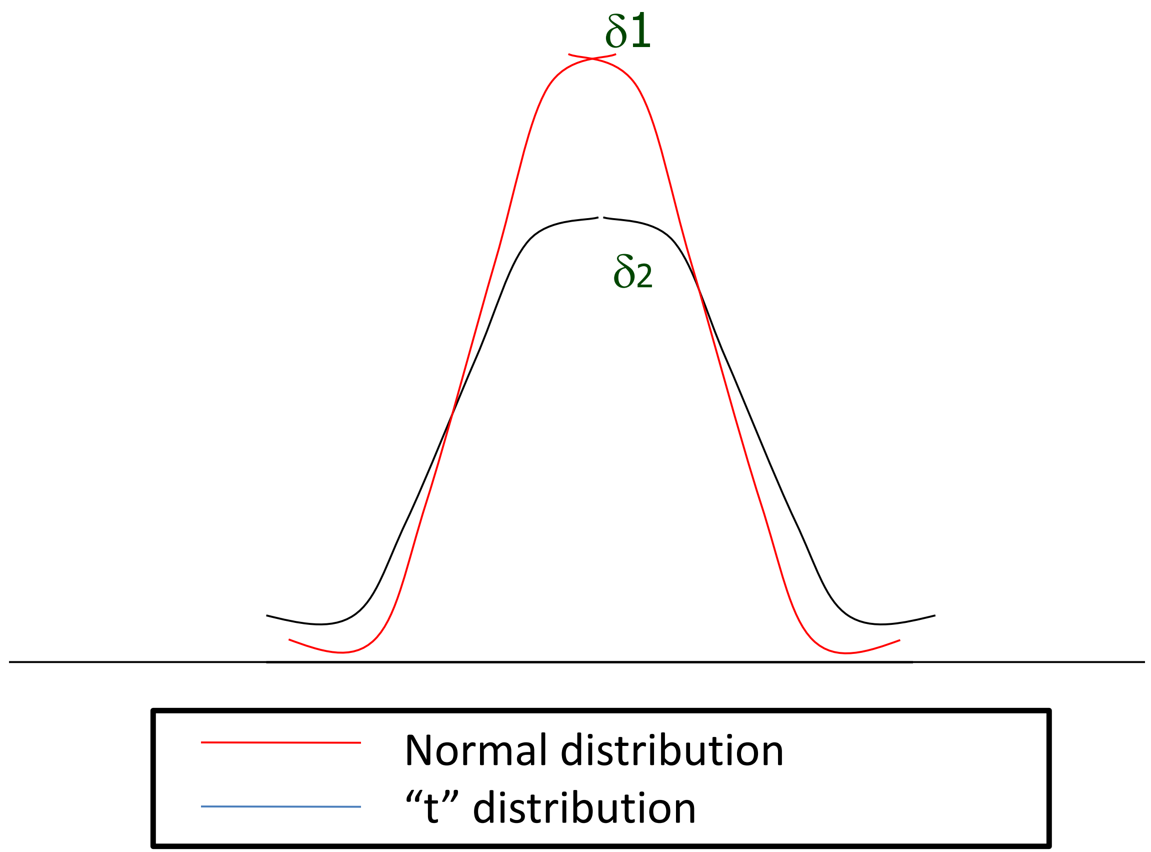
- When n is 30 or more , the two curves are very close.

Normal Distribution



Mean ± 1 SD covers 68.26 % of area under the curve
Mean ± 2 SD covers 95.45 % of area under the curve
Mean ± 3 SD covers 99.74 % of area under the curve


t-distributions
df=4
df=3
df=2
df=1
P
0.4
0.3
0.2
0.1
0
−4
−3
−2
−1
0
1
2
3
4
t


δ1
δ2
Normal distribution
“t” distribution

Difference…

z formula

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

t formula

$$t = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

Assumptions of *t*-test

- Dependent variables are interval or ratio.
- The population from which samples are drawn is normally distributed.
- Samples are randomly selected.
- The groups have equal variance (Homogeneity of variance).
- The t-statistic is robust (it is reasonably reliable even if assumptions are not fully met.)

Applications of *t*-test

- The calculation of a confidence interval for a sample mean.
- To test whether a sample mean is different from a hypothesized value.
- To compare mean of two samples.
- To compare two sample means by group.

Recall.. Hypothesis testing

- To assist administrators and clinicians in making decision
- Statistical test is only one piece of evidences , should not be interpreted as definitive, and consider with other relevant information.

- A statement about one or more population
- Research hypothesis, Statistical hypothesis

Hypothesis:

Null Hypothesis is hypothesis to be tested, designated by H_0

Also called 'Hypothesis of no difference'

Alternative Hypothesis (Research hypothesis) (H_A)

<u>Rule for stating statistical hypothesis:</u>

Can we conclude that a certain pop. mean is not 50?

$H_0 : \mu = 50 \qquad H_A : \mu \neq 50$

Can we conclude that a certain pop. mean greater than 50?

$H_0 : \mu \leq 50 \qquad H_A : \mu > 50$

Can we conclude that a certain pop. mean less than 50?

$H_0 : \mu \geq 50 \qquad H_A : \mu < 50$

Examples of common type of hypothesis testing…..

- Determining the significant difference by comparing a sample to a given population
  - e.g. The mean pulse rate for male adult age between 35-45 is 77/min
- Determining the significance by comparing samples derived from 2 or more population interest
  - e.g. The mean pulse rate between male and female adult is different
- Analyzing the significant relationship between variables

Types of *t*-tests

- Single sample *t-test*
  - *We have only one group*
  - *Want to test again hypothetical mean*
- Independent samples *t-test*
  - *We have 2 means, 2 groups, no relation between groups*
  - *Eg. When we want to compare the mean of Test Treat group with placebo group*
- Paired *t-test*
  - *It consists of samples of matched pairs of similar units or one group of units and tested twice*
  - *Difference of mean between pre & post drug intervention*

One sample *t-test*

- It is used in measuring whether a sample value significantly differs from a hypothesized value.
  - For example, a research scholar might hypothesize that on an average it takes 3 minutes for people to drink a standard cup of coffee.
  - He conducts an experiment and measures how long it takes his subjects to drink a standard cup of coffee.
  - The one sample t-test measures whether the mean amount of time it took the experimental group to complete the task varies significantly from the hypothesized 3 minutes value.

The test statistic for a One Sample *t* Test is denoted *t*, which is calculated using the following formula:

$$t = \frac{\overline{x} - \mu}{s_{\bar{x}}}$$

where

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

where

- μ = Proposed constant for the population mean
- $\bar{x}$ = Sample mean
- n = Sample size (i.e., number of observations)
- s = Sample standard deviation
- $s_{\bar{x}}$ = Estimated standard error of the mean (*s*/sqrt(*n*))

The calculated *t* value is then compared to the critical *t* value from the *t* distribution table with degrees of freedom *df* = *n* - 1 and chosen confidence level. If the calculated *t* value > critical *t* value, then we reject the null hypothesis.

Steps to test a hypothesis about a mean

1. State the null and alternative hypotheses
2. Select the decision criterion (level of significance)
3. Establish the critical value
4. Draw a random sample from a population and calculate the mean of the sample

5. Calculate the standard deviation and SE of sample

6. Calculate the value of the test statistic t that correspond to the mean of the sample

7. Compare the calculated value of t with the critical value of t and then accept or reject the null hypothesis

Example

Research Questions:

-Do incoming students experience more or less stress than sophomores (2^{nd} year students)?

- 16 randomly selected students filled out a standard stress scale.
- Through years of testing, we know that sophomores typically score about 26 on this test (Out of 60).

- Data
  - Hypothesized μ = 26
  - Sample Data
    - n=16
    - S^2 = 5.25
    - $\overline{X}$ = 28

2. **Statistical Hypotheses**:

Null hypothesis : μ =26

Alternative hypothesis: $\mu \neq 26$

3. **Decision rule**

(set α = 0.05 ,two tailed)

df = (n-1)= (16-1)= 15

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	$\boxed{0.05}$	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
$\boxed{15}$	0.000	0.691	0.866	1.074	1.341	1.753	$\boxed{2.131}$	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

Critical value from table ± 2.131

4. **Compute** observed t for one sample mean

$$t = \frac{\overline{X} - \mu}{S_{\overline{X}}}$$

$$S_{\overline{X}} \quad \sqrt{\frac{5.25}{16}} = 0.57$$

$$t = \frac{28 - 26}{0.57} = 3.51$$

5. **Make a decision**

Observed t (3.51) exceed critical value (2.131)

Decision --- Reject Ho

6. **Interpret / Report results**:

- Difference not likely due to chance.
- Incoming students report more stress than the typical level reported by sophomores, $t(15) = 3.51$, $p <$ or equal 0.05, two tailed.

Exercise
One-sample *t*-test

- Research Questions
  - Are first year students average test scores significantly different from the score of 100?
  - Sample Data
    - Mean 107.8 (SD 5.35)
    - N=10
- Please follow the steps in example 1.

Example: t-test using SPSS

- According to the CDC, the mean height of adults ages 20 and older is about 66.5 inches (69.3 inches for males, 63.8 inches for females).
- RQ: Let's test if the mean height of our sample data is significantly different than 66.5 inches using a one-sample *t* test.
- The null and alternative hypotheses of this test will be:
- H_0: 66.5 = μ_{Height} ("the mean height of the sample is equal to 66.5")
  H_1: 66.5 ≠ μ_{Height} ("the mean height of the sample is not equal to 66.5")

One sample *t*-test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Height	408	68.0318	5.32566	.26366

One-Sample Test

	Test Value = 66.5 A					
	B t	C df	D Sig. (2-tailed)	E Mean Difference	95% Confidence Interval of the Difference F	
					Lower	Upper
Height	5.810	407	.000	1.53176	1.0135	2.0501

How to read?

- A **Test Value**: The number we entered as the test value in the One-Sample T Test window.
- B **t Statistic**: The test statistic of the one-sample *t* test, denoted *t*. In this example, t = 5.810. Note that *t* is calculated by dividing the mean difference (E) by the standard error mean (from the One-Sample Statistics box).
- C **df**: The degrees of freedom for the test. For a one-sample *t* test, df = n - 1; so here, df = 408 - 1 = 407.
- D **Sig. (2-tailed)**: The two-tailed p-value corresponding to the test statistic.
- E **Mean Difference**: The difference between the "observed" sample mean (from the One Sample Statistics box) and the "expected" mean (the specified test value (A)). The sign of the mean difference corresponds to the sign of the *t* value (B). The positive *t* value in this example indicates that the mean height of the sample is greater than the hypothesized value (66.5).
- F **Confidence Interval for the Difference**: The confidence interval for the difference between the specified test value and the sample mean.

How to interpret?

- **Decision and Conclusions**
- Since $p < 0.001$, we reject the null hypothesis that the sample mean is equal to the hypothesized population mean and conclude that the mean height of the sample is significantly different than the average height of the overall adult population.
- Based on the results, we can state the following:
- There is a significant difference in mean height between the sample and the overall adult population ($p < .001$).
- The average height of the sample is about 1.5 inches taller than the overall adult population average.

Independent *t*-test

- The Independent Samples t Test compares the means of two independent groups in order to determine whether there is statistical evidence that the associated population means are significantly different.
- The independent sample t-test consists of tests that compare mean value(s) of continuous-level (interval or ratio data), in a normally distributed data.
- The independent samples t-test is also called unpaired t-test/ two sample t test.
- It is the t-test to be used when two separate independent and identically distributed variables are measured.
  - Eg: 1. Comparison of quality of life improved for patients who took drug Valporate as opposed to patients who took drug Levetiracetam in myoclonic seizures.
  - Comparasion of mean cholesterol levels in treatment group with placebo group after administration of test drug.

Assumptions

- A random sample of each population is used.
- The random samples are each made up of independent observation.
- Each sample is independent of one another.
- The population distribution of each population must be nearly normal, or the size of the sample is large.
- Population Variances Are Unknown But Assumed Equal

- The null hypothesis (H_0) and alternative hypothesis (H_1) of the Independent Samples *t* Test can be expressed in two different but equivalent ways:
- H_0: $\mu_1 = \mu_2$ ("the two population means are equal")
  H_1: $\mu_1 \neq \mu_2$ ("the two population means are not equal")

OR

- H_0: $\mu_1 - \mu_2 = 0$ ("the difference between the two population means is equal to 0")
  H_1: $\mu_1 - \mu_2 \neq 0$ ("the difference between the two population means is not 0")

- where μ_1 and μ_2 are the population means for group 1 and group 2, respectively. Notice that the second set of hypotheses can be derived from the first set by simply subtracting μ_2 from both sides of the equation.

Homogeneity of variance

- Recall that the Independent Samples *t* Test requires the assumption of *homogeneity of variance* -- i.e., both groups have the same variance.
- The hypotheses for Levene's test are:
- H_0: $\sigma_1^2 - \sigma_2^2 = 0$ ("the population variances of group 1 and 2 are equal")
  H_1: $\sigma_1^2 - \sigma_2^2 \neq 0$ ("the population variances of group 1 and 2 are not equal")
- This implies that if we reject the null hypothesis of Levene's Test, it suggests that the variances of the two groups are not equal; i.e., that the homogeneity of variances assumption is violated.
- If Levene's test indicates that the variances are equal across the two groups (i.e., *p*-value large), you will rely on the first row of output, **Equal variances assumed**, when you look at the results for the actual Independent Samples *t* Test (under *t*-test for Equality of Means).
- If Levene's test indicates that the variances are not equal across the two groups (i.e., *p*-value small), you will need to rely on the second row of output, **Equal variances not assumed**, when you look at the results of the Independent Samples *t* Test (under the heading *t*-test for Equality of Means).

..the test statistic

EQUAL VARIANCES ASSUMED

When the two independent samples are assumed to be drawn from populations with identical population variances (i.e., $\sigma_1^2 = \sigma_2^2$) , the test statistic t is computed as:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

with

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

Where

- $\bar{x}_1$ = Mean of first sample
- $\bar{x}_2$ = Mean of second sample
- n_1 = Sample size (i.e., number of observations) of first sample
- n_2 = Sample size (i.e., number of observations) of second sample
- s_1 = Standard deviation of first sample
- s_2 = Standard deviation of second sample
- s_p = Pooled standard deviation

- The calculated *t* value is then compared to the critical *t* value from the *t* distribution table with degrees of freedom $df = n_1 + n_2 - 2$ and chosen confidence level. If the calculated *t* value is greater than the critical *t* value, then we reject the null hypothesis.
- Note that this form of the independent samples T test statistic assumes equal variances.
- Because we assume equal population variances, it is OK to "pool" the sample variances (s_p). However, if this assumption is violated, the pooled variance estimate may not be accurate, which would affect the accuracy of our test statistic (and hence, the p-value).

EQUAL VARIANCES NOT ASSUMED

When the two independent samples are assumed to be drawn from populations with unequal variances (i.e., $\sigma_1^2 \neq \sigma_2^2$), the test statistic *t* is computed as:

$$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where

$\bar{x}_1$ = Mean of first sample

$\bar{x}_2$ = Mean of second sample

n_1 = Sample size (i.e., number of observations) of first sample

n_2 = Sample size (i.e., number of observations) of second sample

s_1 = Standard deviation of first sample

s_2 = Standard deviation of second sample

The calculated *t* value is then compared to the critical *t* value from the *t* distribution table with degrees of freedom

- The calculated *t* value is then compared to the critical *t* value from the *t* distribution table with degrees of freedom and chosen confidence level. If the calculated *t* value > critical *t* value, then we reject the null hypothesis.
- Note that this form of the independent samples T test statistic does not assume equal variances. This is why both the denominator of the test statistic and the degrees of freedom of the critical value of *t* are different than the equal variances form of the test statistic.

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}$$

Example: Independent *t*-test using SPSS

- Students reported their typical time to run a mile, and whether or not they were an athlete.
- Suppose we want to know if the average time to run a mile is different for athletes versus non-athletes.
- This involves testing whether the sample means for mile time among athletes and non-athletes in your sample are statistically different (and by extension, inferring whether the means for mile times in the population are significantly different between these two groups).
- You can use an Independent Samples *t* Test to compare the mean mile time for athletes and non-athletes.

- The hypotheses for this example can be expressed as:
- H_0: $\mu_{\text{non-athlete}} - \mu_{\text{athlete}} = 0$ ("the difference of the means is equal to zero")
  H_1: $\mu_{\text{non-athlete}} - \mu_{\text{athlete}} \neq 0$ ("the difference of the means is not equal to zero")
- where μ_{athlete} and $\mu_{\text{non-athlete}}$ are the population means for athletes and non-athletes, respectively.
- In the sample data, we will use two variables: *Athlete* and *MileMinDur*. The variable *Athlete* has values of either "0" (non-athlete) or "1" (athlete). It will function as the independent variable in this T test. The variable *MileMinDur* is a numeric duration variable (h:mm:ss), and it will function as the dependent variable. In SPSS, the first few rows of data look like this:

Group Statistics

	Are you an athlete?	N	Mean	Std. Deviation	Std. Error Mean
Mile time	Non-athlete	226	0:09:06	0:02:01.668	0:00:08.093
	Athlete	166	0:06:51	0:00:49.464	0:00:03.839

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Mile time	Equal variances assumed	102.98	.000	13.475	390	.000	0:02:14	0:00:10	0:01:55	0:02:34
	Equal variances not assumed			15.047	315.846	.000	0:02:14	0:00:08	0:01:57	0:02:32

How to read the result?

- There are two parts that provide different pieces of information: (A) Levene's Test for Equality of Variances and (B) t-test for Equality of Means.
- A **Levene's Test for Equality of of Variances**:
- The *p*-value of Levene's test is printed as ".000" (but should be read as $p < 0.001$ -- i.e., p very small), so we we reject the null of Levene's test and conclude that the variance in mile time of athletes is significantly different than that of non-athletes.
- **This tells us that we should look at the "Equal variances not assumed" row for the *t* test (and corresponding confidence interval) results**.

- B **t-test for Equality of Means** provides the results for the actual Independent Samples *t* Test. From left to right:
- *t* is the computed test statistic
- *df* is the degrees of freedom
- *Sig (2-tailed)* is the p-value corresponding to the given test statistic and degrees of freedom
- *Mean Difference* is the difference between the sample means; it also corresponds to the numerator of the test statistic
- *Std. Error Difference* is the standard error; it also corresponds to the denominator of the test statistic

How to interpret?

- Since p < .001 is less than our chosen significance level α = 0.05, we can reject the null hypothesis, and conclude that the that the mean mile time for athletes and non-athletes is significantly different.
- Based on the results, we can state the following:
  - There was a significant difference in mean mile time between non-athletes and athletes ($t_{315.846}$ = 15.047, p < .001).
  - The average mile time for athletes was 2 minutes and 14 seconds faster than the average mile time for non-athletes.

Paired t test

For related samples

- A paired t-test is used to compare two population means where you have two samples in which observations in one sample can be paired with observations in the other sample.

- A comparison of two different methods of measurement or two different treatments where the measurements/treatments are applied to the same subjects.
  - Eg: 1.pre-test/post-test samples in which a factor is measured before and after an intervention
  - Eg: 2.Cross-over trials in which individuals are randomized to two treatments and then the same individuals are crossed-over to the alternative treatment
  - Eg; 3.Matched samples, in which individuals are matched on personal characteristics such as age and sex
  - Suppose a sample of "n" subjects were given an antihypertensive drug we want to check blood pressure before and after treatment . We want to find out the effectiveness of the treatment by comparing mean pre & post t/t

- The hypotheses can be expressed in two different ways that express the same idea and are mathematically equivalent:
- H_0: $\mu_1 = \mu_2$ ("the paired population means are equal")
  H_1: $\mu_1 \neq \mu_2$ ("the paired population means are not equal")

OR

- H_0: $\mu_1 - \mu_2 = 0$ ("the difference between the paired population means is equal to 0")
  H_1: $\mu_1 - \mu_2 \neq 0$ ("the difference between the paired population means is not 0")
- where
  - μ_1 is the population mean of variable 1, and
  - μ_2 is the population mean of variable 2.

Research question
Does therapy reduce anxiety?
Measure anxiety levels before and after therapy.
One sample(same people) measured twice

Example

Patient	BeforeX_1	After X_2	D= X_{1} - X_2
1	40	24	+16
2	42	30	+12
3	36	37	-1
4	31	21	+10
5	55	32	+23

Transform two scores into one score

Difference score D = $X_1 – X_2$

- Now We have one sample of data, not two’
- Conduct a simple t test on the D’s
- If there is no difference, average D should be =0

Hypotheses in related samples

- Two tailed

$H_o : \mu_D = 0$

$H_A : \mu_D \neq 0$

- One tailed

When $(X_1 < X_2)$ $H_o : \mu_D >$ or $= 0$; $H_A : \mu_D < 0$

When $(X_1 > X_2)$ $H_o : \mu_D <$ *or* $=0$ *:* $H_A : \mu_D > 0$

Test Statistic

The test statistic for the Paired Samples *t* Test, denoted *t*, follows the same formula as the one sample *t* test.

$$t = \frac{\overline{x}_{\text{diff}} - 0}{s_{\overline{x}}}$$

where

$$s_{\overline{x}} = \frac{s_{\text{diff}}}{\sqrt{n}}$$

where

- $\bar{x}_{\text{diff}}$ = Sample mean of the differences
- n = Sample size (i.e., number of observations)
- s_{diff} = Sample standard deviation of the differences
- $s_{\bar{x}}$ = Estimated standard error of the mean (*s*/sqrt(*n*))

The calculated *t* value is then compared to the critical *t* value with df = *n* - 1 from the *t* distribution table for a chosen confidence level. If the calculated *t* value is greater than the critical *t* value, then we reject the null hypothesis (and conclude that the means are significantly different).

Example: Paired *t*-test using SPSS

- The sample dataset has placement test scores (out of 100 points) for four subject areas: English, Reading, Math, and Writing.
- Suppose we are particularly interested in the English and Math sections, and want to determine whether English or Math had higher test scores on average.
- We could use a paired *t* test to test if there was a significant difference in the average of the two tests.

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	English & Math	398	.243	.000

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	English	82.7441	398	6.84480	.34310
	Math	65.4468	398	8.46214	.42417

Paired Samples Test

	Paired Differences					t	df	Sig. (2-tailed)
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
English - Math	17.30	9.50303	.4763	16.3608	18.2337	36.313	397	.000

How to read the result?

- **Paired Samples Statistics** gives univariate descriptive statistics (mean, sample size, standard deviation, and standard error) for each variable entered. Notice that the sample size here is 398; this is because the paired t-test can only use cases that have non-missing values for both variables.
- **Paired Samples Correlations** shows the bivariate Pearson correlation coefficient (with a two-tailed test of significance) for each pair of variables entered.
- **Paired Samples Test** gives the hypothesis test results.

- The Paired Samples Statistics output repeats what we examined before we ran the test. The Paired Samples Correlation table adds the information that English and Math scores are significantly positively correlated (r = .243).

- Reading from left to right:
- First column: The pair of variables being tested, and the order the subtraction was carried out. (If you have specified more than one variable pair, this table will have multiple rows.)
- **Mean:** The average difference between the two variables.
- **Standard deviation:** The standard deviation of the difference scores.
- **Standard error mean:** The standard error (standard deviation divided by the square root of the sample size). Used in computing both the test statistic and the upper and lower bounds of the confidence interval.
- **t:** The test statistic (denoted *t*) for the paired T test.
- **df:** The degrees of freedom for this test.
- **Sig. (2-tailed)**: The *p*-value corresponding to the given test statistic *t* with degrees of freedom *df*.

- **Decision and Conclusions**
- From the results, we can say that:
  - English and Math scores were weakly and positively correlated ($r = 0.243$, $p < 0.001$).
  - There was a significant average difference between English and Math scores ($t_{397} = 36.313$, $p < 0.001$).
  - On average, English scores were 17.3 points higher than Math scores (95% CI [16.36, 18.23]).

Acknowledgement…

- This power point is prepared referring to
  - Power point presentation by Prof Dr Hla Hla Win ("t" distribution and "t" test)
  - Power point presentation by Dr Kyaw Oo (Statistical Methods)
  - Power point presentation by Dr Hla Soe Tint (Statistics)
  - An Introduction to Medical Statistics by Martin Bland
  - Intuitive Biostatistics by Harvey Motulsky
  - Study Design and Statistical Analysis, APracticalGuideforClinicians2006 by Mitchell Katz
  - Medical Statistics at a glance by Aviva Petrie & Caroline Sabin
  - Medical Statistics Made Easy by Michael Harris & Gordon Taylor

Thank You……

Attached… calculation for *t*-test

Group Exercise

Exercise….. 1

- RQ: Will anti-TB treatment improve the weight of TB patients?

- What are the dependent and independent variables?
  - xxx
  - xxx

- What type of *t*-test has to be used?
  - xxx

Exercise….. 2

- RQ: Are the Body Mass Index of urban reproductive aged unmarried women different from that of rural?

- What are the dependent and independent variables?
  - xxx
  - xxx

- What type of *t*-test has to be used?
  - xxx

Exercise….. 3

- RQ: Is there difference between the peak flow rates among asthma patients treated with new-bronchodilator and those treated with the placebo?

- What are the dependent and independent variables?
  - xxx
  - xxx

- What type of *t*-test has to be used?
  - xxx

Exercise….. 4

- RQ: Is there difference between mean age of Diabetics and Non-Diabetics?

- What are the dependent and independent variables?
  - xxx
  - xxx

- What type of *t*-test has to be used?
  - xxx

Exercise..... 5

- Please interpret the t-test result (output from SPSS)

Independent t-test

- Mean difference between weights of European and Japanese cars

Group Statistics

	ORIGIN Country of Origin	N	Mean	Std. Deviation	Std. Error Mean
WEIGHT Vehicle Weight (lbs.)	2 European	73	2431.49	490.884	57.454
	3 Japanese	79	2221.23	320.497	36.059

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
WEIGHT Vehicle Weight (lbs.)	Equal variances assumed	18.242	.000	3.150	150	.002	210.27	66.756	78.362	342.169
	Equal variances not assumed			3.100	122.367	.002	210.27	67.832	75.990	344.541

Paired *t*-test

- Mean difference between current salary and beginning salary among one sample

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Current Salary	$34,419.57	474	$17,075.661	$784.311
	Beginning Salary	$17,016.09	474	$7,870.638	$361.510

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper			
Pair 1	Current Salary - Beginning Salary	$17,403.481	$10,814.620	$496.732	$16,427.407	$18,379.555	35.036	473	.000

Unpaired *t*-test

- Mean difference between R's occupational prestige score among male and female

Group Statistics

	Respo nde...	N	Mean	Std. Deviation	Std. Error Mean
R's Occupational Prestige Score (1980)	Male	621	43.85	13.278	.533
	Female	797	42.22	12.864	.456

Independent Samples Test

		t-test for Equality of Means						
							95% Confidence Interval of the Difference	
		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
R's Occupational Prestige Score (1980)	Equal variances assumed	2.343	1416	.019	1.636	.698	.267	3.006
	Equal variances not assumed	2.334	1311.942	.020	1.636	.701	.261	3.012

Exercise….. 6 (Home work)

- Please read the distributed paper (will send via email), and discuss the followings.
  - What are the objectives of study?
    - xxx
  - What are the study population?
    - xxx
  - What types of *t*-test is used? And Why?
    - xxx
  - What does the *t*-test result answer for the study objective?
    - xxx

Exercise… 7 (Home work)

- Please answer the distributed multiple choice questions. (will send via email, answer keys will be included).